

SHDOM

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1. INTRODUCTION

SHDOM is a general purpose, publicly available, three-dimensional atmospheric radiative transfer model. SHDOM is an explicit method, which means it solves for the whole radiation field, as distinct from Monte Carlo methods which solve for particular radiative outputs. SHDOM is particularly well suited for remote sensing applications, where it can compute outgoing radiances at many angles from a cloud field at virtually no extra cost. SHDOM is not appropriate for calculating domain average quantities for which Monte Carlo methods excel.

The I3RC intercomparison offers an opportunity to explore the pros and cons of SHDOM and Monte Carlo models on some real world inhomogeneous cloud fields. Specifically, we wish to determine the computer resources required to achieve a particular accuracy for a certain number of outputs using SHDOM and Monte Carlo models. This will help guide modelers on the appropriate choice of SHDOM or Monte Carlo for their applications. To emphasize the importance of this accuracy versus CPU time tradeoff, we are submitting two SHDOM entries (low and high resolution) in the I3RC.

2. SHDOM OVERVIEW

The SHDOM algorithm is described in Evans (1998). SHDOM efficiently discretizes the radiation field by representing the source function at grid points with an adaptive spherical harmonic expansion in the angular variables. The solution method is to transform the source function to discrete ordinates and calculate radiances by integrating the radiative transfer equation along the discrete ordinates. The radiances are then transformed back to spherical harmonics where they are used to compute the source function. The number of such iterations required increases with the optical thickness and the single scattering albedo. During the solution iterations new grid points may be created by splitting

cells according to the local gradient in the source function. The angular resolution is specified by the number of discrete ordinates in the zenith angle direction N_μ and in the azimuthal direction N_ϕ (for 3D problems there is a total of $N_\mu N_\phi$ discrete ordinates). The N_μ and N_ϕ also control the maximum number of spherical harmonic terms. The spatial resolution is controlled by the resolution of the “base” grid ($N_x \times N_y \times N_z$ grid points) and a parameter that determines the amount of cell splitting (splitacc).

3. CASE 2: MONTE CARLO COMPARISON

One method of determining the accuracy of SHDOM is to compare with an independent method of radiative transfer solution. For the radar derived cloud field (case 2) we show a comparison between SHDOM and a Monte Carlo model. This forward Monte Carlo model uses the maximal cross section method, and was operated in a mode that bilinearly interpolates when sampling the extinction field. Thus the Monte Carlo model makes the same assumption that SHDOM does about the extinction being defined at grid points and interpolated in between. The Monte Carlo model has been found to agree, within the noise, with a code developed by A. Marshak. The I3RC case 2 grid cell optical depths were translated to grid point extinctions by interpolating the cells above and below. This assures that the column optical depths are preserved. The Monte Carlo model currently does not output radiances, so only fluxes are compared.

To eliminate Monte Carlo noise as a concern, the model was run with 1.6×10^8 photons and 1000 orders of scattering. We estimate the pixel level rms flux error to be less than 0.002. The Monte Carlo CPU times are very large, but these may be easily scaled to larger errors (e.g. 8700 sec for 0.005 accuracy in experiment 1). Table 1 lists the Monte Carlo and SHDOM CPU times for the 8 experiments.

The two SHDOM entries are: **high** ($N_\mu = 12, N_\phi = 24, N_x = 640, N_z = 55$, splitacc=0.01) and **low** ($N_\mu = 6, N_\phi = 12, N_x = 320, N_z = 28$, splitacc=0.02). The number of SHDOM iterations

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Table 1: Comparison of the Monte Carlo and two resolution SHDOM CPU times. The forward Monte Carlo run has 1.6×10^8 photons. The SHDOM high resolution run has $N_\mu = 12$, $N_x = 640$ while the low resolution run has $N_\mu = 6$, $N_x = 320$. Experiments 1-5 have a Henyey-Greenstein phase function, while 6-8 have a C1 phase function.

| Exp | SZA | ω | A_s | CPU seconds | | |
|-----|-----|----------|-------|-------------|------|-----|
| | | | | MC | high | low |
| 1 | 0 | 1.00 | 0.0 | 54396 | 597 | 54 |
| 2 | 60 | 1.00 | 0.0 | 50111 | 627 | 55 |
| 3 | 0 | 0.99 | 0.0 | 54451 | 473 | 47 |
| 4 | 60 | 0.99 | 0.0 | 50353 | 532 | 44 |
| 5 | 60 | 1.00 | 0.4 | 110565 | 737 | 61 |
| 6 | 0 | 1.00 | 0.0 | 53289 | 467 | 44 |
| 7 | 60 | 1.00 | 0.0 | 50508 | 477 | 50 |
| 8 | 60 | 1.00 | 0.4 | 111013 | 600 | 51 |

range from 28 to 36 for a solution accuracy of 10^{-4} . The high resolution CPU times are typically 10 minutes, while the low resolution ones are about 1 minute. These CPU times are for a 450 MHz Pentium II computer with 512 MB memory running Linux. A Portland Group Fortran 90 compiler was used for SHDOM. A similar machine, the Dell Precision 410 with the Intel compiler has a SPECfp95 base of 11.8.

An example of the Monte Carlo and SHDOM fluxes is shown in Fig. 1. The SHDOM and MC fluxes agree very well in this most realistic and difficult 2D case. The downwelling flux disagreement around $X = 18$ km may be due to this area having nonzero extinction at the flux reporting level of 0.63 km, which means that SHDOM interpolates some optical depth below that level.

The absolute pixel level rms flux differences are given in Table 2. There is significant variation in SHDOM error between experiments, but overall the high resolution case has about twice the accuracy of the low resolution case (e.g. 0.0036 vs. 0.0067 for upwelling flux). Even though the pixel level SHDOM errors do not average out the way Monte Carlo noise does, it is interesting to look at the domain average flux comparison in Table 3. The downwelling flux is consistently overestimated by SHDOM by about 0.004 for the high resolution case and by about 0.010 for the low resolution runs. There is considerably more error cancellation for mean upwelling flux, which has much smaller errors than the rms upwelling flux difference. We expect the I3RC SHDOM-Monte Carlo differences to be larger due

Case 2: SHDOM/Monte Carlo Flux Comparison

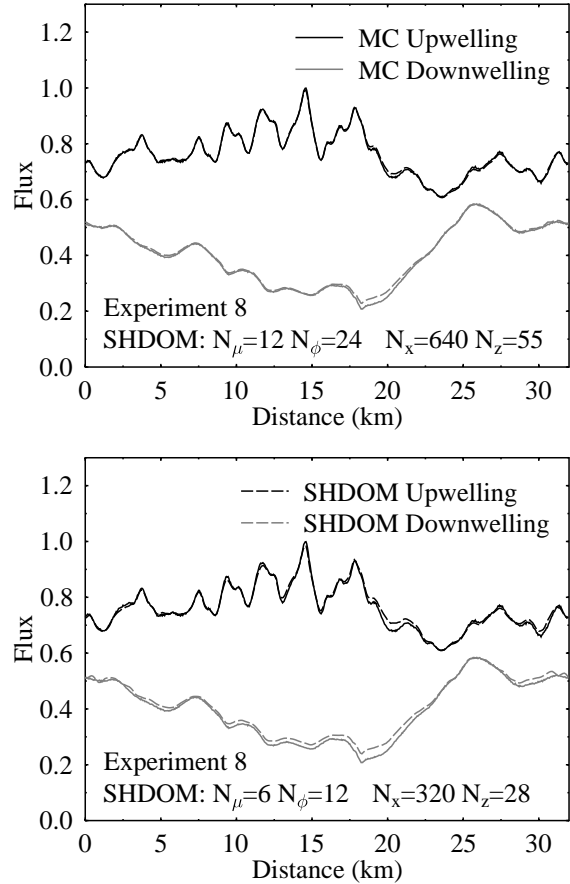


Figure 1: Comparison of outgoing fluxes calculated with SHDOM and a Monte Carlo model for the 2D case 2 field. The two resolutions of SHDOM submitted to the I3RC are shown. The fluxes are for experiment 8, which had the largest disagreement.

to the usual way Monte Carlo models represent the extinction field.

4. CASE 3: SHDOM CONVERGENCE TEST

For the 3D Landsat cloud field (case 3) we performed a convergence test to estimate the SHDOM accuracy. This involves running a *reference* SHDOM case at high angular and spatial resolution. The pixel level fluxes and radiances for lower resolution SHDOM runs are then compared to the reference case using rms differences. The resources required for the very high resolution reference case dictate that the convergence test must be done in only two dimensions. We chose an X-Z slice at

Table 2: Pixel level rms flux differences between Monte Carlo and SHDOM high and low resolutions runs for the 8 case 2 experiments.

| Exp | high | | low | |
|-----|--------------|----------------|--------------|----------------|
| | $F \uparrow$ | $F \downarrow$ | $F \uparrow$ | $F \downarrow$ |
| 1 | 0.0022 | 0.0052 | 0.0031 | 0.0105 |
| 2 | 0.0056 | 0.0061 | 0.0094 | 0.0114 |
| 3 | 0.0014 | 0.0044 | 0.0030 | 0.0102 |
| 4 | 0.0041 | 0.0043 | 0.0072 | 0.0104 |
| 5 | 0.0050 | 0.0076 | 0.0095 | 0.0142 |
| 6 | 0.0023 | 0.0043 | 0.0035 | 0.0103 |
| 7 | 0.0034 | 0.0058 | 0.0086 | 0.0118 |
| 8 | 0.0050 | 0.0075 | 0.0096 | 0.0148 |
| Avg | 0.0036 | 0.0057 | 0.0067 | 0.0117 |

Table 3: Domain mean fluxes and flux differences (SHDOM - Monte Carlo) for SHDOM high and low resolutions runs for the 8 case 2 experiments.

| Exp | Upwelling Flux | | | Downwelling Flux | | |
|-----|----------------|---------------|--------------|------------------|---------------|--------------|
| | MC | high Δ | low Δ | MC | high Δ | low Δ |
| 1 | .5593 | .0008 | .0012 | .4407 | .0042 | .0084 |
| 2 | .6978 | -.0003 | .0020 | .3022 | .0040 | .0097 |
| 3 | .4019 | -.0003 | .0020 | .3071 | .0034 | .0089 |
| 4 | .5520 | -.0010 | .0019 | .2004 | .0038 | .0095 |
| 5 | .7580 | .0003 | .0022 | .4033 | .0048 | .0114 |
| 6 | .5613 | -.0006 | .0012 | .4387 | .0034 | .0088 |
| 7 | .7016 | .0007 | .0018 | .2984 | .0051 | .0102 |
| 8 | .7607 | -.0003 | .0022 | .3988 | .0048 | .0121 |

$Y=0.7$ km as representative, including the highest optical depth and clear regions. The reference case has $N_\mu = 32$, $N_\phi = 64$, $N_x = 1024$, $N_z = 321$, and no cell splitting. The convergence test was performed only for the conservative scattering experiments.

Table 4 shows some of the convergence test results. Cases with the adaptive grid cell splitting are not shown as they often (embarrassingly) increased the error. This may be due to the vertically uniform extinction which is conducive to a regular grid. For a given angular resolution, there is a large decrease in error in going from $N_z = 41$ to $N_z = 81$, which makes nearly square grid cells. The convergence test and subsequent 3D SHDOM runs were made on an SGI Origin with 250 MHz IP27 R10000 processors and a SPECfp rating of 23.2.

A summary of all the convergence test results, including both sun angles and cell splitting, is shown in Fig. 2. The range of CPU times for a given accu-

Table 4: Selected SHDOM convergence test results for the 2D Landsat slice with $\theta_0 = 60^\circ$ and $\omega = 1$. rms differences from the reference case are divided by the mean. The reference case had $N_\mu = 32$, $N_x = 1024$, $N_z = 321$. The CPU times are in seconds on an SGI Origin.

| N_μ | N_x | N_z | CPU | rms errors / mean | | | |
|---------|-------|-------|------|-------------------|----------------|--------------|----------------|
| | | | | $I \uparrow$ | $I \downarrow$ | $F \uparrow$ | $F \downarrow$ |
| 4 | 64 | 41 | 6 | 0.073 | 0.073 | 0.067 | 0.077 |
| 4 | 128 | 81 | 16 | 0.051 | 0.074 | 0.075 | 0.073 |
| 6 | 64 | 41 | 9 | 0.061 | 0.041 | 0.032 | 0.037 |
| 6 | 128 | 81 | 29 | 0.023 | 0.017 | 0.018 | 0.018 |
| 8 | 128 | 41 | 23 | 0.068 | 0.052 | 0.052 | 0.055 |
| 8 | 128 | 81 | 43 | 0.010 | 0.010 | 0.013 | 0.012 |
| 8 | 256 | 161 | 246 | 0.010 | 0.009 | 0.013 | 0.010 |
| 12 | 128 | 41 | 44 | 0.072 | 0.059 | 0.060 | 0.064 |
| 12 | 128 | 81 | 101 | 0.005 | 0.008 | 0.008 | 0.008 |
| 16 | 128 | 41 | 85 | 0.073 | 0.062 | 0.062 | 0.066 |
| 16 | 128 | 81 | 197 | 0.005 | 0.006 | 0.006 | 0.007 |
| 16 | 256 | 161 | 1367 | 0.004 | 0.003 | 0.004 | 0.003 |

racy illustrates how there are many suboptimal parameter choices when running SHDOM. The lower envelope of CPU time versus radiance error is approximately the best performance of SHDOM *for this particular problem*. That is, one cannot necessarily extrapolate this to other situations, though we will from 2D to 3D. The slope of the lower envelope is about -1.0 to -1.5. This is surprisingly good performance, given that this is a four dimensional problem ($N_\mu \times N_\phi \times N_x \times N_z$). By contrast, the Monte Carlo CPU time would be expected to depend on the error to the -2.0 power.

Based on the 2D convergence test, we decided to submit 3D results for $N_\mu = 12$, $N_\phi = 24$, $N_x = 128$, $N_y = 128$, $N_z = 81$ for the high resolution entry and $N_\mu = 6$, $N_\phi = 12$, $N_x = 128$, $N_y = 128$, $N_z = 81$ for the low resolution entry. Table 5 lists the SGI Origin CPU times for the four experiments and two SHDOM resolutions. The low resolution experiments take under 6 hours to produce 128×128 radiance fields at many directions with an estimated 2% accuracy.

5. DISCUSSION AND CONCLUSIONS

SHDOM is far more efficient than Monte Carlo models at computing many radiative quantities from small scale inhomogeneous cloud fields. By small scale, we mean those where the grid spacing is comparable to the mean free path, so the radiative trans-

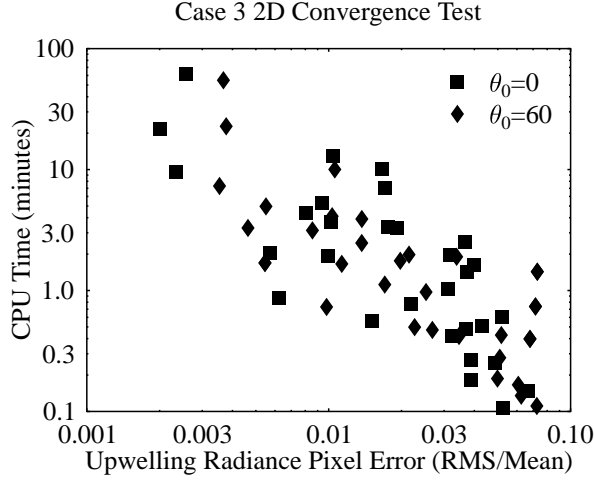


Figure 2: The Landsat field convergence test CPU time as a function of radiance accuracy.

Table 5: The SHDOM CPU times for the 3D Landsat cloud field. The SHDOM high resolution run has $N_\mu = 12$ while the low resolution run has $N_\mu = 6$.

| Exp | SZA | ω | CPU seconds | |
|-----|-----|----------|-------------|-------|
| | | | high | low |
| 1 | 0 | 1.00 | 114594 | 19475 |
| 2 | 60 | 1.00 | 91057 | 16449 |
| 3 | 0 | 0.99 | 82839 | 21351 |
| 4 | 60 | 0.99 | 82184 | 21453 |

fer is resolved. SHDOM should, therefore, be an important tool for remote sensing applications. Monte Carlo models are generally more efficient and accurate than SHDOM at computing domain averaged quantities. The I3RC should enable us to detail the tradeoff between CPU time, accuracy, and number of radiative outputs for Monte Carlo and SHDOM.

We propose parameterizing the CPU time for remote sensing applications in terms of the number of radiances output and the desired accuracy. Assume we have a $N_x \times N_y \times N_z$ 3D grid, where the number of vertical grid cells N_z is proportional to optical depth. Let the desired number of radiance directions be N_{dir} and the rms error be ϵ . The SHDOM CPU time could be expressed as

$$\text{CPU}_{\text{SHDOM}} = a_S N_x N_y N_z^{b_S} \epsilon^{-c_S}$$

and the Monte Carlo CPU time could be parame-

terized as

$$\text{CPU}_{\text{MC}} = a_M N_{dir} N_x N_y N_z^{b_M} \epsilon^{-2}.$$

The SHDOM CPU time increases as $N_z^{b_S}$ with $1 < b_S < 2$ because SHDOM generally needs grid cell optical depths that are small and because the number of iterations increases with optical depth. The Monte Carlo CPU time increases as $N_z^{b_M}$ with $1 < b_M < 2$ because the maximal cross section virtual mean free path is the inverse of the maximum extinction and the photon path length also increases with optical depth. The major differences between these two CPU time expressions is that the Monte Carlo one depends on the number of radiance directions and the prefactor a_M is usually larger than a_S .

SHDOM is considerably more difficult to use and to estimate the accuracy characteristics for. We showed two methods to estimate the accuracy of SHDOM for particular problems. One is comparison to Monte Carlo results with very low pixel noise. Another is a 2D convergence test comparing SHDOM at various resolutions to a very high resolution reference case. We believe that there are substantial improvements that could be made to SHDOM in terms of accuracy and ease of use. However, part of the difficulty in using SHDOM is due to its flexibility to handle most types of atmospheric radiative transfer calculations.

The phase 1 I3RC cloud cases have been designed for the usual Monte Carlo model framework. The fields are specified as uniform extinction in discrete cells, whereas SHDOM models the field as continuous between grid points. Comparisons of SHDOM and Monte Carlo results in the I3RC will therefore give *larger differences than those shown here* (because the Monte Carlo model used here also assumes a continuous field). The I3RC single scattering albedo and phase functions are uniform across the domain, while SHDOM can handle the more realistic situation of optical properties varying with location (as effective radius changes). The radiances are to be output only for the zenith and nadir directions because that is particularly efficient for some Monte Carlo models. SHDOM is much more flexible than this as illustrated with the two Landsat cloud images shown in Fig. 3. Once the solution iterations are finished, SHDOM can compute radiances for many directions at very little additional cost.

On the other hand, there are many modeling scenarios for which SHDOM is ill-suited. For example, large scale cloud fields from a cloud resolving model with 1 km grid cells have large optical paths across a grid cell. These resolutions do not resolve the radia-

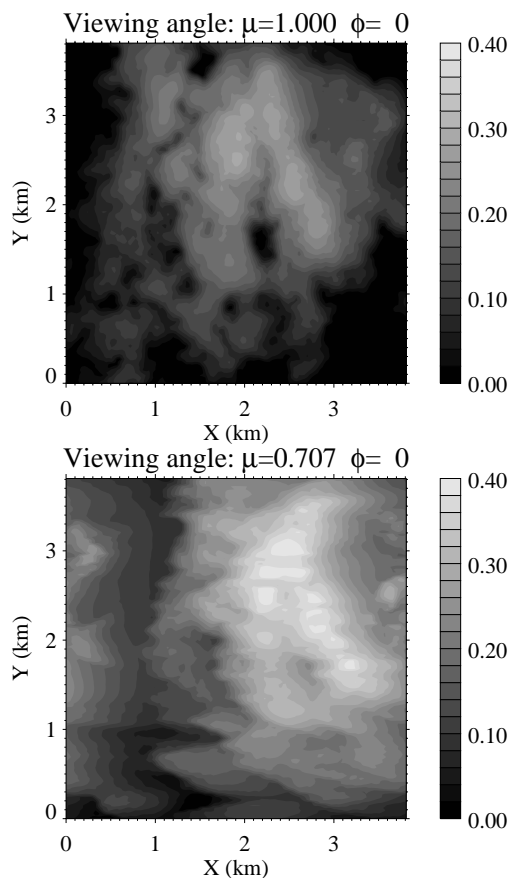


Figure 3: Two views of the upwelling reflectance from the Landsat field (case 3). These are but two of the 109 angles at which the upwelling radiance was computed.

tive transfer (i.e., the mean free path is much smaller than a cell), and SHDOM would take too many resources to be feasible. These large scale scenes are appropriate for climate issues, where the focus is on broadband domain average fluxes over a GCM grid box. Broadband Monte Carlo models would be the tool of choice for large scale scenes.

The remote sensing cases in future I3RC phases should use cloud fields at a scale that resolves the radiative transfer (i.e. cells 10 – 100 m). It is well known (e.g. Loeb and Coakley, 1998) that subpixel (for AVHRR) variability can cause significant effects on pixel radiances. Therefore, it would be inappropriate to assume that AVHRR-scale pixels are homogeneous. Future I3RC remote sensing cases could include molecular absorption, molecular Rayleigh scattering, and aerosols with realistic boundary layer

cloud fields generated with large eddy simulations. A range of viewing and solar zenith angles should be considered. The goals could include showing the wider remote sensing community that 3D cloud effects are important and that we have the general purpose tools to accurately simulate these effects.

REFERENCES

- Evans, K. F., 1998: The spherical harmonics discrete ordinate method for three-dimensional atmospheric radiative transfer. *J. Atmos. Sci.*, **55**, 429–446.